

# QCD Analysis of Deep Inelastic Diffractive Scattering at HERA

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## Abstract

The QCD analysis of deep inelastic diffractive scattering is performed assuming the dominance of the "soft" pomeron exchange and simple, physically motivated parametrization of parton distributions in a pomeron. Results of the analysis are compared with the recent data obtained by the H1 collaboration at HERA. Both the LO and NLO approximations are considered and the theoretical predictions concerning the quantity  $R = F_L^D/F_T^D$  for diffractive structure functions are presented.

The diffractive deep inelastic scattering is the process

$$e + p \rightarrow e' + X + p' \quad (1)$$

where there is a large rapidity gap between the diffractively produced system  $X$  and the recoiled proton  $p'$  (or excited proton state). To be precise, the process (1) reflects the diffractive interaction of the virtual photon (see Fig. 1a). The recent measurements performed by both ZEUS and H1 collaborations at the DESY ep collider HERA [1, 2, 3] have shown that the diffractive structure function constitutes a significant part of the proton structure function  $F_2$ .

The cross section for the process (1) is related in the following way to the diffractive structure functions  $\frac{dF_2^D(x_P, \beta, Q^2, t)}{dx_P dt}$ ,  $\frac{dF_L^D(x_P, \beta, Q^2, t)}{dx_P dt}$ :

$$\frac{d\sigma^D}{dx dQ^2 dx_P dt} = \frac{4\pi^2 \alpha^2}{x Q^4} \left[ \left( 1 - y + \frac{y^2}{2} \right) \frac{dF_2^D(x_P, \beta, Q^2, t)}{dx_P dt} - \frac{y^2}{2} \frac{dF_L^D(x_P, \beta, Q^2, t)}{dx_P dt} \right] \quad (2)$$

where the kinematical variables are defined below:

$$\begin{aligned}
s &= (p_e + p)^2 \approx 2p_e p & y &= \frac{pq}{p_e p} \\
Q^2 &= -q^2 & x &= \frac{Q^2}{2pq} \\
t &= (p - p')^2 & \beta &= \frac{Q^2}{2q(p - p')} & x_P &= \frac{x}{\beta}
\end{aligned} \tag{3}$$

and where  $q = p_e - p'_e$ . The four momenta  $p_e, p'_e, p$  and  $p'$  correspond to those of the initial electron, final electron, initial proton and the recoiled proton respectively (see Fig. 1).

The main aim of this paper is to analyse the diffractive processes in deep inelastic  $ep$  scattering assuming that they are dominated by a "soft" pomeron exchange [4]- [10]. Different description of this processes based on the "hard" pomeron which follows from perturbative QCD has been discussed in [11] -[14]. The "hard" pomeron would however lead to much steeper  $x_P$  dependence than that which is observed experimentally [3].

Assuming that the diffractive process  $\gamma^*(Q^2) + p \rightarrow X + p$  is dominated by the pomeron exchange (see Fig. 1b) with the pomeron being described as a Regge pole with its trajectory

$$\alpha_P(t) = \alpha_P(0) + \alpha'_P t \tag{4}$$

where for the "soft" pomeron  $\alpha_P(0) \approx 0.08$ ,  $\alpha'_P = 0.25 \text{ GeV}^{-2}$ , we get the following factorizable expression for the diffractive structure functions :

$$\frac{dF_{2,L}^D(x_P, \beta, Q^2, t)}{dx_P dt} = f(x_P, t) F_{2,L}^P(\beta, Q^2, t) \tag{5}$$

The function  $f(x_P, t)$  is the so called "pomeron flux factor" which, if the diffractively recoiled system is a single proton, has the following form [10]:

$$f(x_P, t) = N x_P^{1-2\alpha_P(t)} \frac{B^2(t)}{16\pi} \tag{6}$$

where  $B(t)$  describes the pomeron coupling to a proton. Its normalization is such that the pomeron contribution to the  $pp$  total cross-section  $\sigma_{pp}(s)$  is

$$\sigma_{pp}(s) = B^2(0) \left( \frac{s}{s_0} \right)^{\alpha_P(0)-1} \tag{7}$$

with  $s_0 = 1 \text{ GeV}^2$ . The normalization factor  $N$  was set to be equal to  $\frac{2}{\pi}$  following the convention of refs. [4, 6, 10]. We have also set [10]

$$B(t) = 4.6 \text{ mb}^{1/2} \exp(1.9 \text{ GeV}^{-2} t) \tag{8}$$

and have slightly increased  $\alpha_P(0)$  in the flux factor  $f(x_P, t)$  (6) up to  $\alpha_P(0) = 1.1$  [3]. The slope of the pomeron trajectory was kept equal to  $\alpha'_P = 0.25 \text{ GeV}^{-2}$ .

The functions  $F_{2,L}^P(\beta, Q^2, t)$  are the pomeron structure functions with the variable  $\beta$  playing the role of the Bjorken scaling variable for the  $\gamma^*(Q^2)$  pomeron inelastic "scattering". The factorization property of the diffractive structure function into the pomeron "flux" and the pomeron structure function is the direct consequence of the single pomeron exchange mechanism of diffraction and of the assumption that the pomeron is described by a single Regge pole. Factorization does not in general hold in models which are based entirely on perturbative QCD [10, 15]. Recent measurements [3] have however found that the factorization of the diffractive structure functions agrees very well with the experimental data. It has moreover been established that the  $x_P$  dependence of the flux factor (integrated over  $t$ ) can be parametrized as  $x_P^{-n}$  with  $n = 1.19$ . This observation confirms dominance of the soft pomeron exchange since the "hard" pomeron with its intercept  $\alpha_P(0) \approx 1.5$  would give much steeper behaviour  $\sim x_P^{-2}$  or so.

In the region of large  $Q^2$  the pomeron structure function  $F_2^P$  is expected to be described by the "hand-bag" diagram (see Fig. 2a) [4, 6]. It provides the partonic description of deep inelastic diffraction which leads to the Bjorken scaling mildly violated by the logarithmic scaling violations implied by perturbative QCD. In the QCD improved parton model the pomeron structure function  $F_2^P(\beta, Q^2, t)$  is related in conventional way to the quark  $q_i^P(\beta, Q^2, t)$  distributions in a pomeron:

$$F_2^P(\beta, Q^2, t) = 2 \beta \sum_i e_i^2 q_i^P(\beta, Q^2, t) \quad (9)$$

where  $e_i$  are the quark charges (note that  $q_i^P(\beta, Q^2, t) = \bar{q}_i^P(\beta, Q^2, t)$ ). The formula (9) acquires  $O(\alpha_s)$  corrections in the next-to-leading approximation of perturbative QCD. The next-to-leading approximation is also needed for theoretically consistent introduction of the pomeron longitudinal structure function  $F_L^P(\beta, Q^2, t)$ . In what follows we shall therefore discuss both the leading and next to leading approximations. In the leading order approximation we shall set  $F_L^P = 0$ .

At first we shall specify the details of the parton distributions in the pomeron at the reference scale  $Q^2 = Q_0^2$  which we set equal to  $4\text{GeV}^2$ .

At small  $\beta$  both the quark and gluon distributions are assumed to be dominated by the pomeron exchange as illustrated in the Fig. 2b.

$$\begin{aligned} \beta q_i^P(\beta, Q_0^2, t) &= a_i(t) \beta^{1-\alpha_P(0)} \\ \beta g^P(\beta, Q_0^2, t) &= a_g(t) \beta^{1-\alpha_P(0)} \end{aligned} \quad (10)$$

The functions  $a_i(t)$  and  $a_g(t)$  can be estimated from the factorization of pomeron couplings [4, 7, 9] (see Fig. 2b):

$$\begin{aligned} a_i(t) &= r(t) a_i^p \\ a_g(t) &= r(t) a_g^p \end{aligned} \quad (11)$$

where the parameters  $a_i^p$  and  $a_g^p$  are the pomeron couplings controlling the normalization of the small  $x$  behaviour of the sea quark and gluon distributions in a proton i.e.

$$\begin{aligned} xq_i^p(x, Q_0^2) + x\bar{q}_i^p(x, Q_0^2) &= 2 a_i^p x^{1-\alpha_P(0)} \\ xg^p(x, Q_0^2) &= a_g^p x^{1-\alpha_P(0)} \end{aligned} \quad (12)$$

and the function  $r(t)$  is:

$$r(t) = \frac{\pi}{2} \frac{G_{PPP}(t)}{B(0)} \quad (13)$$

The coupling  $G_{PPP}(t)$  is the triple pomeron coupling (see Fig.2 b) and its magnitude can be estimated from the cross-section of the diffractive production  $p + \bar{p} \rightarrow p + X$  in the limit of large mass  $M_X$  of the diffractively produced system  $X$ . The relevant formula for this cross-section is:

$$s \frac{d\sigma}{dt dM_X^2} = \frac{B^2(t)}{16\pi} G_{PPP}(t) B(0) \left( \frac{s}{M_X^2} \right)^{2\alpha_P(t)-1} \left( \frac{M_X^2}{s_0} \right)^{\alpha_P(0)-1} \quad (14)$$

(The factor  $\frac{\pi}{2}$  in the eq. (13) follows from the convention  $N = \frac{2}{\pi}$  for the normalization factor  $N$  in the eq. (6) [4, 6, 10]). We neglected the (weak)  $t$  dependence of the function  $r(t)$  and have estimated its magnitude from the recent Tevatron data [16] as  $r(t) \approx r(0) = 0.089$ . The parameters  $a_i^p$  were estimated assuming that the sea quark distributions in a proton can be parametrized as:

$$xq_i^p(x, Q_0^2) + x\bar{q}_i^p(x, Q_0^2) = 2 a_i^p x^{1-\alpha_P(0)} (1-x)^7 \quad (15)$$

and fixing the constants  $a_i^p$  from the requirement that the average momentum fraction which corresponds to those distributions is the same as that which follows from the recent parametrization of parton distributions in a proton [17, 18]. The momentum sum rule has also been used to fix the parameter  $a_g^p$  i.e. we assumed

$$xg^p(x, Q_0^2) = a_g^p x^{1-\alpha_P(0)} (1-x)^5 \quad (16)$$

and imposed the condition that the gluons carry 1/2 momentum of the proton. We extrapolated the pomeron dominated quark and gluon distributions in a pomeron (see (10)) to the region of arbitrary values of  $\beta$  by multiplying the factor  $\beta^{1-\alpha_P(0)}$  by  $1-\beta$  [7].

We have also included the term proportional to  $\beta(1-\beta)$  in both the quark and gluon distributions [7]. The normalization of this term in the quark distributions has been estimated in [6] assuming that it is dominated by the quark-box diagram of Fig.2c with the non-perturbative couplings of pomeron to quarks. In this model one gets:

$$\beta q^P(\beta, Q_0^2, t) = \frac{C\pi}{3} \beta (1-\beta) \quad (17)$$

where  $C \approx 0.17$  [6]. We found that the fairly reasonable description of data can be achieved provided that the constant  $C$  is enhanced by a factor equal to 1.5. We have also assumed that the relative normalization of the quark distributions in a pomeron corresponding to different flavours is the same as that of the sea quark distributions in

a proton [17, 18]. Finally the normalization of the term proportional to  $\beta(1 - \beta)$  in the gluon distribution in a pomeron has been obtained by imposing the momentum sum rule.

As the result of the estimates and extrapolations discussed above the parametrization of parton distributions in a pomeron at the reference scale  $Q_0^2 = 4\text{GeV}^2$  looks as follows:

$$\begin{aligned}\beta u^P(\beta, Q_0^2, t) &= 0.4 (1 - \delta) S^P(\beta) \\ \beta d^P(\beta, Q_0^2, t) &= 0.4 (1 - \delta) S^P(\beta) \\ \beta s^P(\beta, Q_0^2, t) &= 0.2 (1 - \delta) S^P(\beta) \\ \beta c^P(\beta, Q_0^2, t) &= \delta S^P(\beta) \\ \beta g^P(\beta, Q_0^2, t) &= (0.218 \beta^{-0.08} + 3.30 \beta) (1 - \beta)\end{aligned}\tag{18}$$

where the function  $S^P(\beta)$  is parametrized as below

$$S^P(\beta) = (0.0528 \beta^{-0.08} + 0.801 \beta) (1 - \beta)\tag{19}$$

and  $\delta=0.02$  [17, 18]. Following the approximations discussed above we have neglected the  $t$  dependence in those parton distributions. The analysis of the pomeron structure functions based on different parametrizations of parton distributions in a pomeron has recently been presented in refs. [9, 19].

The parton distributions defined by eqs.(18,19) were next evolved up to the values of  $Q^2$  for which the data exist using the LO Altarelli-Parisi evolution equations [20, 21] with  $\Lambda = 0.255 \text{ GeV}$  [18]. In Fig. 3a,b we show our results for the quantity:

$$F_2^D(\beta, Q^2) = \int_{x_{PL}}^{x_{PH}} dx_P \int_{-\infty}^0 dt \frac{dF_2^D(x_P, \beta, Q^2, t)}{dx_P dt}\tag{20}$$

with  $x_{PL} = 0.0003$  and  $x_{PH} = 0.05$  [3]. The theoretical curves are compared with the recent data from the H1 collaboration at HERA [3]. We find that the agreement with the data is very good concerning both the shape in  $\beta$  as well as the evolution with  $Q^2$ . The following points should be noticed:

- The diffractive structure function  $F_2^D(\beta, Q^2)$  at the moderate values of  $\beta$  is dominated by "hard" component of the quark distributions in a pomeron (i.e. that component which is proportional to  $\beta(1 - \beta)$ ). Relatively small magnitude of the soft component (i.e. of the term which is proportional to  $\beta^{-0.08}$ ) is a direct consequence of the small magnitude of the triple pomeron coupling (see the eqs. (10),(11), (18) and (19)).
- The soft component of the parton distributions is important at the small values of  $\beta \leq 0.1$  or so. The agreement of our parametrization with the data shows that the estimate of its normalization from the factorization of pomeron couplings is quite reasonable. At large  $Q^2$  the  $\beta$  spectrum becomes softer as the result of the QCD evolution.

- It turns out that gluons carry more than 60 % of the pomeron momentum. The gluon content of the pomeron can only indirectly manifest itself through the QCD evolution of the diffractive structure function with  $Q^2$ . It should be observed at this point that the relatively large gluon distributions in a pomeron with a "hard"  $1 - \beta$  spectrum leads to the increase of the diffractive structure functions with increasing  $Q^2$  for fixed  $\beta$ , up to rather relatively large values of  $\beta \approx 0.5$ . It may be seen from Fig. 3a that this effect is nicely confirmed by the data. It should be noted at this point that the proton structure function  $F_2^p(x, Q^2)$  starts to decrease with increasing  $Q^2$  already for  $x \geq 0.1$ .

We have also performed the NLO QCD analysis in order to be able to consistently introduce the quantity  $R = F_L^D(\beta, Q^2)/F_T^D(\beta, Q^2)$  where  $F_T^D(\beta, Q^2) = F_2^D(\beta, Q^2) - F_L^D(\beta, Q^2)$  and

$$F_L^D(\beta, Q^2) = \int_{x_{PL}}^{x_{PH}} dx_P \int_{-\infty}^0 dt \frac{dF_L^D(x_P, \beta, Q^2, t)}{dx_P dt} \quad (21)$$

We compare the NLO and LO results for  $F_2^D(\beta, Q^2)$  in Fig. 4 a,b. We can see that both approximations lead to similar predictions. In Fig. 4 c,d we show our results for  $R$ . The longitudinal diffractive structure function is mainly driven by the gluon distributions in a pomeron and a large amount of gluons (see 18) implies that  $R$  can reach 0.5 for  $\beta \leq 0.1$ . It may also be seen from Figs. 4 a,b that  $R$  exhibits much more softer dependence on  $\beta$  than the structure function  $F_2^D(\beta, Q^2)$ .

To summarize, we have presented in this paper the QCD analysis of the diffractive structure function within the simple soft pomeron exchange picture. In this model the diffractive structure functions factorize into the flux factor and the pomeron structure functions which are sensitive on the parton content of the pomeron. We based our analysis on the simple parametrization of the parton distributions which utilized both the factorization of pomeron couplings and the quark-box contribution and obtained a good description of the experimental data. We have also performed the NLO QCD analysis and estimated the magnitude of  $R = F_L^D(\beta, Q^2)/F_T^D(\beta, Q^2)$  (see Fig. 4 c,d). Its knowledge should be useful for more refined experimental determination of the diffractive structure function  $F_2^D$ .

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## Figure Captions

1. (a) Kinematics of the large rapidity gap process  $e + p \rightarrow e' + X + p'$ . The wavy line represents the virtual photon.  
 (b) The Pomeron exchange diagram for diffractive production of the hadronic system  $X$  by a virtual photon. The wavy and zigzag lines represent the virtual photon and pomeron respectively.
2. (a) The hand bag diagram for the virtual Compton diffractive production. The wavy and zigzag lines represent the virtual photon and pomeron respectively. The continuous lines in the upper part of the diagram denote quarks (antiquarks).  
 (b) The triple pomeron diagram contribution to the "hand-bag" diagram of Fig. 2a. The coupling  $G_{PPP}$  denotes the triple pomeron coupling.  
 (c) The quark box diagram contribution to the "hand-bag" diagram of Fig. 2a.
3. Theoretical predictions for the diffractive structure function  $F_2^D(\beta, Q^2)$  defined by the formula (20) and their comparison with the data from HERA [3]. The structure function  $F_2^D(\beta, Q^2)$  is plotted (a) as the function of  $Q^2$  for fixed values of  $\beta$  and (b) as the function of  $\beta$  for fixed values of  $Q^2$ .
4. (a) Comparison of LO (continuous lines) with NLO (dashed lines) results for the structure function  $F_2^D(\beta, Q^2)$  defined by the eq. (20) plotted as the function of  $Q^2$  for fixed values of  $\beta$ .  
 (b) Comparison of LO (continuous lines) with NLO (dashed lines) results for the structure function  $F_2^D(\beta, Q^2)$  defined by the eq. (20) plotted as the function of  $\beta$  for fixed values of  $Q^2$ .
5. (a) Theoretical predictions for the quantity  $R = \frac{F_L^D}{F_T^D}$  plotted as the function of  $Q^2$  for fixed values of  $\beta$ .  
 (b) Theoretical predictions for the quantity  $R = \frac{F_L^D}{F_T^D}$  plotted as the function of  $\beta$  for fixed values of  $Q^2$ .



Figure 3

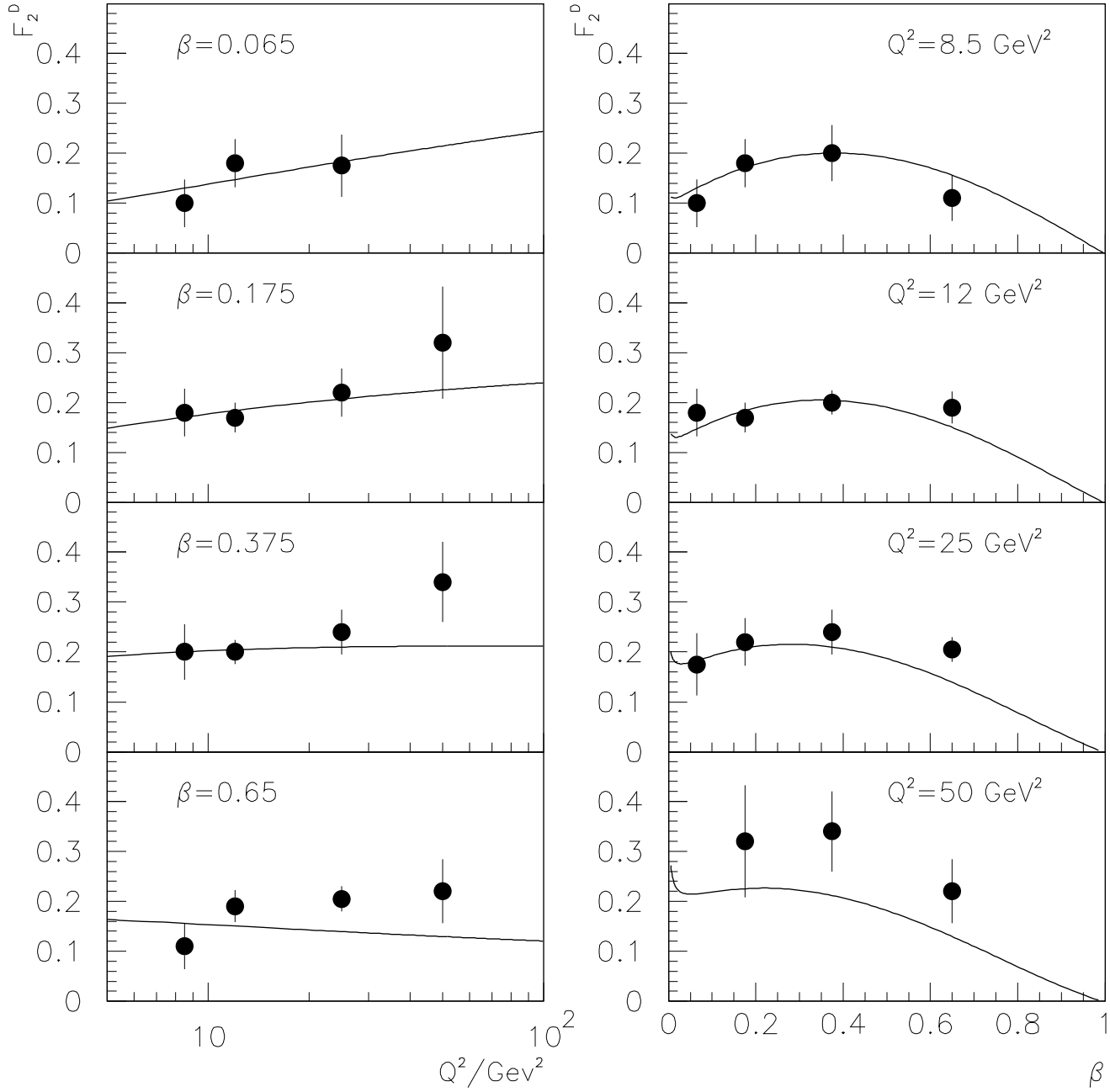


Figure 4

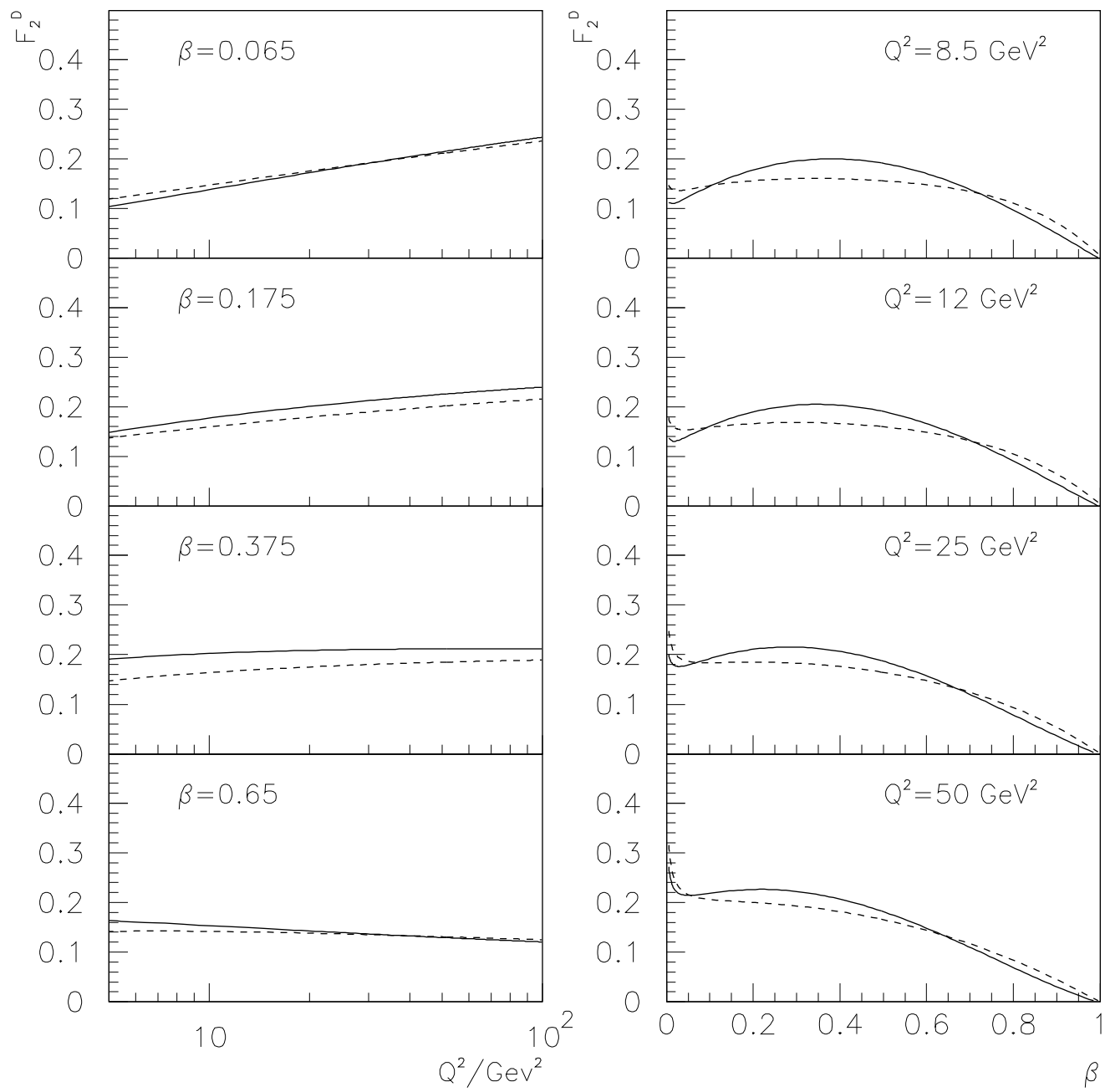
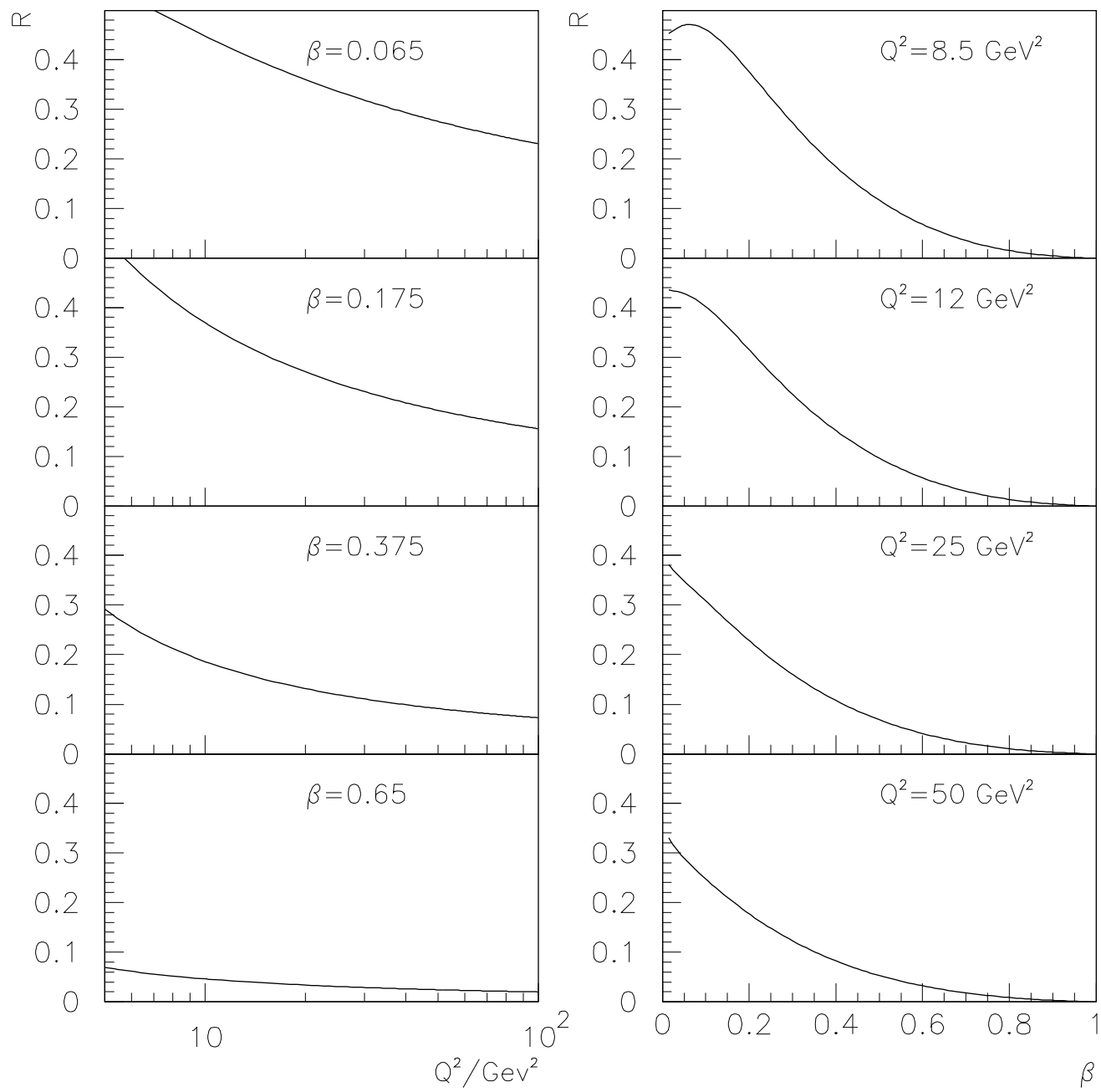


Figure 5



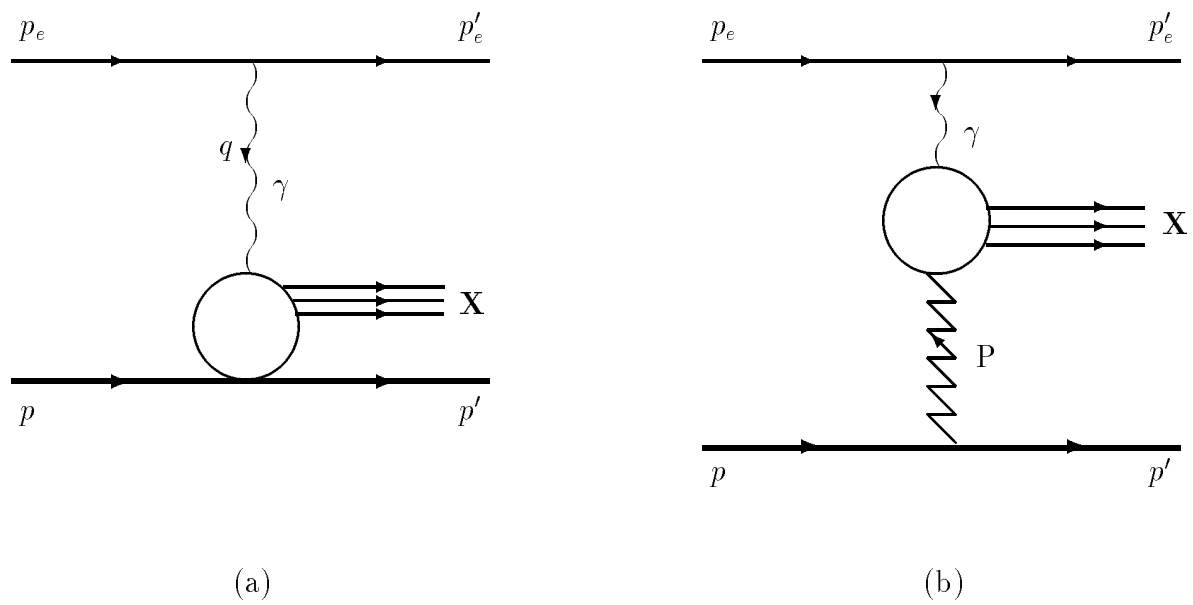


Figure 1

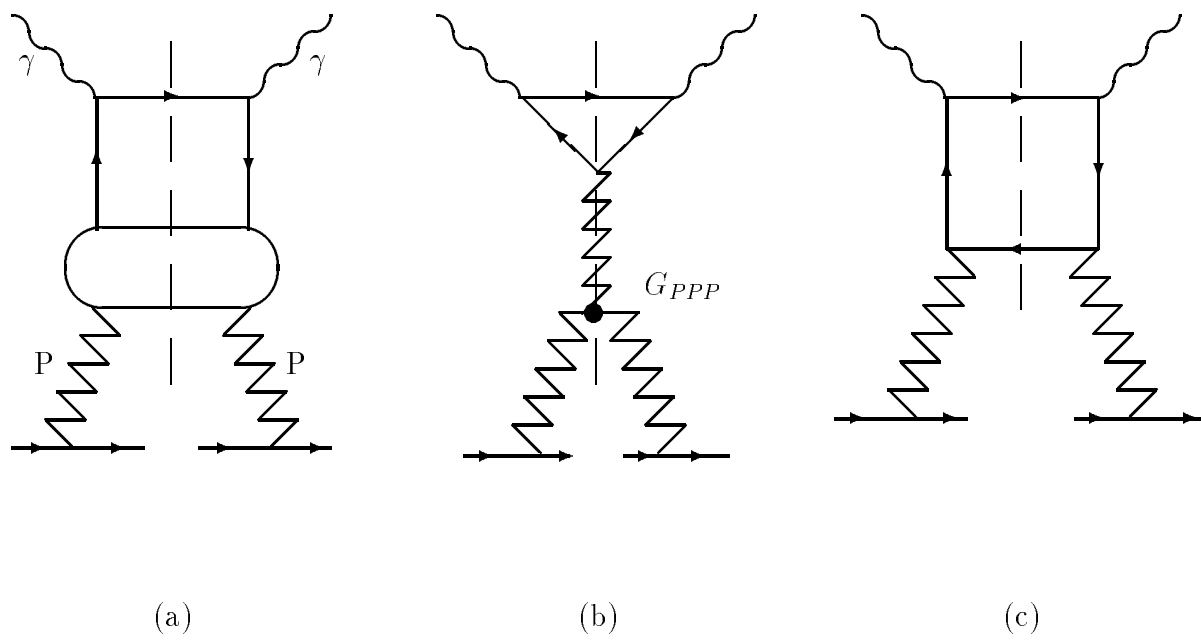


Figure 2